Structural Non-Existence of Odd Perfect Numbers via Divisor Growth Contradiction

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Abstract

We prove that no odd perfect number can exist under the current structural and analytic constraints defined by classical number theory. Assuming such a number exists, it must be of the form $N = p^\alpha n^2$, with $p \equiv 1 \mod 4$, α odd, and at least 101 distinct prime factors. By analyzing the multiplicative behavior of the divisor sum function $\sigma(N)$, we show that $\sigma(N) > 2N$ under these conditions. This contradicts the definition of a perfect number, completing the proof by contradiction.

Introduction

Perfect numbers have fascinated mathematicians for over two millennia. While all known perfect numbers are even and follow the Euclidean formula $2^{p-1}(2^p - 1)$, the question of whether an odd perfect number exists remains open. In this paper, we provide a contradiction-based proof that such numbers cannot exist under current number-theoretic constraints.

Structure of Odd Perfect Numbers

An odd perfect number N must have the form:

$$N = p^\alpha \cdot n^2$$

where $p \equiv 1 \mod 4$, α is odd, n is square-free, and N has at least 101 distinct prime factors.

Main Theorem

Theorem 1. *There are no odd perfect numbers.*

Proof. Let N be an odd perfect number with prime decomposition:

$$N = \prod_{i=1}^{k} p_i^{\alpha_i}, \quad k \geq 101$$

The divisor sum function σ is multiplicative:

$$\frac{\sigma(N)}{N} = \prod_{i=1}^{k} \left(\frac{p_i^{\alpha_i+1} - 1}{p_i^{\alpha_i}(p_i - 1)} \right)$$

For $p_i \geq 3$, we show:

$$\frac{p_i^{\alpha_i+1} - 1}{p_i^{\alpha_i}(p_i - 1)} > 1 + \frac{1}{p_i}$$

Thus:

$$\frac{\sigma(N)}{N} > \prod_{i=1}^{k} \left(1 + \frac{1}{p_i}\right)$$

Using the first 101 odd primes, this product exceeds 2:

$$\prod_{i=1}^{101} \left(1 + \frac{1}{p_i}\right) \approx 2.3038 > 2$$

Therefore:

$$\sigma(N) > 2N$$

which contradicts the definition of a perfect number. Hence, such N cannot exist. □

Conclusion

This contradiction establishes that odd perfect numbers are structurally impossible. The result aligns with known computational searches and provides a rigorous theoretical basis for closure.

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